PARENTESI DI POISSON

$H(q,p)$ $\begin{cases} \frac{dq}{dt} = \frac{\partial H}{\partial p} \\ \frac{dp}{dt} = -\frac{\partial H}{\partial q} \end{cases}$

$F(q,p,t)$ grandezza meccanica (coord. generalizzate e velocità generalizz.)

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\partial F}{\partial q}\cdot\frac{dq}{dt} + \frac{\partial F}{\partial p}\cdot\frac{dp}{dt} = \frac{\partial F}{\partial t} + \boxed{\frac{\partial F}{\partial q}\cdot\frac{\partial H}{\partial p} - \frac{\partial F}{\partial p}\cdot\frac{\partial H}{\partial q}} = \frac{\partial F}{\partial t} + \{F,H\}$$

Sostituisco senza guardare: è il vantaggio delle derivate di Lie.

Per Def. data $F(q,p,t)$ $H(q,p,t)$
chiamo parentesi di Poisson tra F e H

$$\{F,H\} = \frac{\partial F}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial F}{\partial p}\cdot\frac{\partial H}{\partial q}$$

Teo. $\frac{dF}{dt} = \frac{\partial F}{\partial t} + \{F,H\}$

n-gradi di libertà: $H(q_i,p_i)$ $\begin{cases} \frac{dq_i}{dt} = \frac{\partial H}{\partial p_i} \\ \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i} \end{cases}$ (← variabili coniugate)

$F(q_i,p_i,t)$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \sum_{i=1}^{n}\left(\frac{\partial F}{\partial q_i}\frac{dq_i}{dt} + \frac{\partial F}{\partial p_i}\frac{dp_i}{dt}\right) =$$
$$= \frac{\partial F}{\partial t} + \sum_{i=1}^{n}\left(\frac{\partial F}{\partial q_i}\frac{\partial H}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial H}{\partial q_i}\right) =$$
$$= \frac{\partial F}{\partial t} + \{F,H\}$$

ATTENZIONE! al segno: Landau definisce $\{F,H\} = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial p_i}\frac{\partial H}{\partial q_i} - \frac{\partial F}{\partial q_i}\frac{\partial H}{\partial p_i}\right)$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \{H,F\}$$

Prop. formali:

Oss. se $F_1 = q_k$ $\quad \frac{dq_k}{dt} = \{q_k,H\} = \sum_{i=1}^{n}\left(\overset{\delta_{ik}}{\frac{\partial q_k}{\partial q_i}}\frac{\partial H}{\partial p_i} - \cancelto{0}{\frac{\partial q_k}{\partial p_i}\frac{\partial H}{\partial q_i}}\right) = \frac{\partial H}{\partial p_k}$

se $F_2 = p_k$ $\quad \frac{dp_k}{dt} = \{p_k,H\} = \sum_{i=1}^{n}\left(\cancelto{0}{\frac{\partial p_k}{\partial q_i}\frac{\partial H}{\partial p_i}} - \underset{\delta_{ik}}{\frac{\partial p_k}{\partial p_i}}\frac{\partial H}{\partial q_i}\right) = -\frac{\partial H}{\partial q_k}$

Date $f,g$ funz. $C^\infty$ di $(q_i,p_i,t)$

$$\{F,G\} = \sum_i \frac{\partial F}{\partial q_i}\frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p_i}\frac{\partial G}{\partial q_i}$$ (così $\{F,G\}$ è una legge di comp. interna a $C^\infty(\mathbb{R})$)

Prop: 1) $\{F,G\} = -\{G,F\}$ $\forall F,G$ funzioni

2) (Grazie all'antisimmetria è suff. mostrare la lin. per 1 entr.)
$\forall F,\ \alpha_1 G_1 + \alpha_2 G_2 \quad \alpha_1,\alpha_2 \in \mathbb{R}$

$$\{F, \alpha_1 G_1 + \alpha_2 G_2\} = \alpha_1\{F,G_1\} + \alpha_2\{F,G_2\}$$

③ Proprietà di Leibniz

$$\forall F, \ \forall G_1, G_2 \qquad \{F, G_1 G_2\} = G_1\{F, G_2\} + G_2\{F, G_1\}$$
$$= G_1\{F, G_2\} + \{F, G_1\} G_2$$

Oss. $A, B_1, B_2 \in \mathcal{L}(\mathbb{R}^n)$ $\quad [A, B] =: AB - BA$ commutatore di due matrici (difetto di commutatività)

$$[A, B_1 B_2] = A_1 B_1 B_2 - B_1 B_2 A =$$
$$= A_1 B_1 B_2 - B_1 B_2 A - B_1 A_1 B_2 + B_1 A_1 B_2$$
$$= (A_1 B_1 - B_1 A_1) B_2 + B_1 (A_1 B_2 - B_2 A_1)$$
$$= [A, B_1] B_2 + B_1 [A_1, B_2]$$

Dim. $\{F, G_1 G_2\}$ in 1 grado di libertà

$$\{F, G_1 G_2\} = \frac{\partial F}{\partial q} \cdot \frac{\partial G_1 G_2}{\partial p} - \frac{\partial F}{\partial p} \frac{\partial G_1 G_2}{\partial q} =$$
$$= \frac{\partial F}{\partial q}\left(\frac{\partial G_1}{\partial p} G_2 + G_1 \frac{\partial G_2}{\partial p}\right) - \frac{\partial F}{\partial p}\left(\frac{\partial G_1}{\partial q} G_2 + G_1 \frac{\partial G_2}{\partial q}\right)$$
$$= G_2\left(\frac{\partial F}{\partial q}\frac{\partial G_1}{\partial p} - \frac{\partial F}{\partial p}\frac{\partial G_1}{\partial q}\right) + G_1\left(\frac{\partial F}{\partial q} \cdot \frac{\partial G_2}{\partial p} - \frac{\partial F}{\partial p}\frac{\partial G_2}{\partial q}\right)$$
$$= G_2\{F, G_1\} + G_1\{F, G_2\}$$

④ Prop. di Jacobi (permutazioni cicliche di $F, G, H$)

$$\{\{F, G\}, H\} + \{\{G, H\}, F\} + \{\{H, F\}, G\} = 0 \qquad \forall F, H, G$$

Oss. $[[A, B], C] + [[B, C], A] + [[C, A], B] = 0$

$$[AB]C - C[AB] + [BC]A - A[BC] + [CA]B - B[CA] =$$
$$= [AB]C + B[AC] \dots = 0$$

Oss Dall'analogia formale se $\{f, g\} = 0$ commutano secondo Poisson

Parentesi di Poisson e costanti del moto

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \{F, H\}$$

$$\begin{cases} \dfrac{dq_i}{dt} = \dfrac{\partial H}{\partial p_i} \\ \dfrac{dp_i}{dt} = -\dfrac{\partial H}{\partial q_i} \end{cases} \qquad \text{Eq. di Hamilton}$$

$F$ è c.d.m. sse $\frac{\partial F}{\partial t} + \{F, H\} = 0$; se $F$ non dipende esp. da $t$ $\frac{\partial F}{\partial t} = 0$

$F$ c.d.m. $\iff \{F, H\} = 0$

Oss. $\frac{dH}{dt} = \frac{\partial H}{\partial t} + \underset{0}{\underset{|||}{\{H, H\}}}$ $\quad$ (come Jacobi ma con segno cambiato)

per l'antisimmetria

(H è energia, $\frac{\partial H}{\partial t} = 0 \Rightarrow$ quindi H si conserva)

Considero $F, G \ldots$ che non dipendono esplicitamente da $t$
(anche $H$ $\frac{\partial H}{\partial t} = 0$)

① $H$ è c.d.m.
② se $F$ e $G$ sono c.d.m.

$\forall \alpha_1, \alpha_2 \in \mathbb{R}$ $\quad \alpha_1 F + \alpha_2 G$ è c.d.m. (prop. lineare di par. di Poisson)
$F \cdot G$ è c.d.m.

$\{F, G\}$ è una costante del moto (Teor. di Poisson)

Dim. $F, G$ c.d.m. $\Rightarrow$ $\{F, H\} = 0$, $\{G, H\} = 0$

$$\begin{cases} \frac{dq^i}{dt} = \frac{\partial H}{\partial p_i} \\ \frac{dp}{dt} = -\frac{\partial H}{\partial q} \end{cases}$$

$$\{FG, H\} \overset{\text{Leib.}}{=} F \cancel{\{G, H\}}^{0} + G \cancel{\{F, H\}}^{0} = 0$$

$$0 \overset{?}{=} \{\{F, G\}, H\} = -\{\cancel{\{G, H\}}^{0}, F\} - \{\cancel{\{H, F\}}^{0}, G\} = 0$$

Ex. Sia $q_1\ q_2\ q_3$
$x, y, z$
$p_x\ p_y\ p_z$

$L_z = x p_y - y p_x$
$L_y = z p_x - x p_z$
$L_x = y p_z - z p_y$

Prop. Se $L_x$ e $L_y$ è c.d.m. $\Rightarrow$ $L_z$ è cost. del moto
Lemma $L_z = \{L_x, L_y\}$
$(L_k = \varepsilon^{kij} \{L_i L_j\})$

Def. Parentesi di Poisson fondamentali

$$\left[\begin{array}{c|c} \overset{0}{\{q_i, q_j\}} & \{q_i, p_j\} \\ \hline \{p_i, q_j\} & \underset{0}{\{p_i, p_j\}} \end{array}\right] \quad \{q_i, p_j\} = \delta_{ij}$$

$$\{F, G\} = \sum_i \frac{\partial F}{\partial q^i}\frac{\partial G}{\partial p_i} - \frac{\partial F}{\partial p^i}\frac{\partial G}{\partial q^i}$$

$\Rightarrow$ se $F, G$ sono $q_i$, allora $\frac{\partial}{\partial p} = 0$

$$\Rightarrow \sum_k \left( \frac{\partial q_i}{\partial q_k}\frac{\partial p_j}{\partial p^k} - \underbrace{\frac{\partial q_i}{\partial p^k}\frac{\partial p_j}{\partial q^k}}_{0} \right) = \delta_{ij}$$

$$\Rightarrow \{p_i, q_j\} = -\{q_j, p_i\} = -\{q_i, p_j\} = -\delta_{ij}$$

$$\left[\begin{array}{c|c} \mathbb{0}_n & \mathbb{1}_n \\ \hline -\mathbb{1}_n & \mathbb{0}_n \end{array}\right]$$

$\Rightarrow$ $\{q_i, p_j\} = \delta_{ij}$ $\quad (\{p_j, q_i\} = -\delta_{ij})$
$\{q_i, q_j\} = 0$
$\{p_i, p_j\} = 0$

Ex. $\{L_x, L_y\} = \{y p_z - z p_y , z p_x - p_z x\} =$

$$= \{y p_z, z p_x\} - \underbrace{\cancel{\{y p_z, p_z x\}}}_{0} - \underbrace{\cancel{\{z p_y, z p_x\}}}_{0} + \underbrace{\cancel{\{z p_y, p_z x\}}}_{x p_y}$$

$$\{y p_z, z p_x\} = y \{p_z, z p_x\} + \{y, z p_x\} p_z =$$

$$= y z \underbrace{\cancel{\{p_z, p_x\}}}_{0} + y p_x \{\underbrace{p_z, z}_{-1}\} + \underbrace{\cancel{\{y, p_x\}}}_{0} z p_z + \underbrace{\cancel{\{y, z\}}}_{0} p_x p_z$$

perché non sono coniugate

$$= - y p_x$$

$$\{L_x, L_y\} = x p_y - y p_x = L_z$$

Mi

# TRASFORMAZIONI CANONICHE

<u>Ex.</u>
$$\begin{cases} Q_1 = -p_2 + q_1 + 2q_1^2 q_2 \\ Q_2 = p_1 - q_2 - 2q_1 q_2^2 \\ P_1 = p_1 - 2q_1 q_2^2 \\ P_2 = -p_2 + 2q_1^2 q_2 \end{cases}$$

a] I specie

$S(q_1, q_2, Q_1, Q_2)$

$$p_1 = \frac{\partial S}{\partial q_1} \qquad p_2 = \frac{\partial S}{\partial q_2} \qquad P_1 = -\frac{\partial S}{\partial Q_1} \qquad P_2 = -\frac{\partial S}{\partial Q_2}$$

$$\begin{cases} p_1 = Q_2 + q_2 + 2q_1 q_2^2 = \frac{\partial S}{\partial q_1} & \text{IV} \\ p_2 = -Q_1 + q_1 + 2q_1^2 q_2 = \frac{\partial S}{\partial q_2} & \text{III} \\ P_1 = Q_2 + q_2 = -\frac{\partial S}{\partial Q_1} & \text{II} \\ P_2 = Q_1 - q_1 = -\frac{\partial S}{\partial Q_2} & \text{I} \end{cases}$$

I $\quad \dfrac{\partial S}{\partial Q_2} = q_1 - Q_1$

$$S = q_1 Q_2 - Q_1 Q_2 + f(q_1, q_2, Q_1)$$

In generale se non ho mostrato che la trasf. è canonica, il sistema non è compatibile.
Verifico se vale Schwartz (uguaglianza delle der. miste)
CN e S almeno localmente.
In gen. il sistema non ha sol.

II $\quad \dfrac{\partial S}{\partial Q_1} = -q_2 - Q_2 \implies -\cancel{Q_2} + \dfrac{\partial f}{\partial Q_1}(q_1, q_2, Q_1) = -q_2 - \cancel{Q_2}$

$$f(q_1, q_2, Q_1) = -q_2 Q_1 + g(q_1, q_2)$$

$\text{I} \wedge \text{II} \implies S = q_1 Q_2 - Q_1 Q_2 - q_2 Q_1 + g(q_1, q_2)$

III $\quad \dfrac{\partial S}{\partial q_2} = -Q_1 + q_1 + 2q_1^2 q_2$

$$-\cancel{Q_1} + \frac{\partial g}{\partial q_2}(q_1, q_2) = -\cancel{Q_1} + q_1 + 2q_1^2 q_2$$

$$g(q_1, q_2) = q_1 q_2 + q_1^2 q_2^2 + h(q_1)$$

$\text{I} \wedge \text{II} \wedge \text{III} \implies S = q_1 Q_2 - Q_1 Q_2 - q_2 Q_1 + q_1^2 q_2^2 + h(q_1) + q_1 q_2$

IV $\quad \dfrac{\partial S}{\partial q_1} = Q_2 + q_2 + 2q_1 q_2^2$

$$\cancel{Q_2} + \cancel{2q_1 q_2^2} + \cancel{q_2} + \frac{\partial h}{\partial q_1} = \cancel{Q_2} + \cancel{q_2} + \cancel{2q_1 q_2^2} \implies \frac{\partial h}{\partial q_1} = 0$$

$h = 0$ (scelgo la cost. $= 0$)

$$S = q_1 Q_2 - Q_1 Q_2 - q_2 Q_1 + q_1^2 q_2^2 + q_1 q_2$$

b] II specie

$S(q^1, q^2, P_1, P_2)$

$p_1 = \frac{\partial S}{\partial q_1} \qquad Q_1 = \frac{\partial S}{\partial P_1}$

$p_2 = \frac{\partial S}{\partial q_2} \qquad Q_2 = \frac{\partial S}{\partial Q_2}$

(non è nec. che ammetta, una trasf. canonica, una generatrice di ogni specie!)

$$\begin{cases} Q_1 = -p_2 + q_1 + 2q_1^2 q_2 \\ Q_2 = p_1 - q_2 - 2q_1 q_2^2 \\ P_1 = p_1 - 2q_1 q_2^2 \\ P_2 = -p_2 + 2q_1^2 q_2 \end{cases}$$

$$\begin{cases} p_1 = P_1 + 2q_1 q_2^2 & & \overset{?}{=} \frac{\partial S}{\partial q_1} & \text{IV} \\ p_2 = -P_2 + 2q_1^2 q_2 & & = \frac{\partial S}{\partial q_2} & \text{III} \\ Q_1 = P_2 - \cancel{2q_1^2 q_2} + q_1 + \cancel{2q_1^2 q_2} & = P_2 + q_1 & = \frac{\partial S}{\partial P_1} & \text{II} \\ Q_2 = P_1 + \cancel{2q_1 q_2^2} - q_2 - \cancel{2q_1 q_2^2} & = P_1 - q_2 & = \frac{\partial S}{\partial P_2} & \text{I} \end{cases}$$

I $\quad \frac{\partial S}{\partial P_2} = P_1 - q_2 \implies S = P_1 P_2 - q_2 P_2 + f(q_1, q_2, P_1)$

II $\quad \frac{\partial S}{\partial P_1} = P_2 + q_1 \implies \cancel{P_2} + \frac{\partial f}{\partial P_1}(q_1, q_2, P_1) = \cancel{P_2} + q_1$

$f(q_1, q_2, P_1) = q_1 P_1 + g(q_1, q_2)$

I $\wedge$ II $\quad S = P_1 P_2 - q_2 P_2 + q_1 P_1 + g(q_1, q_2)$

III $\quad \frac{\partial S}{\partial q_2} = -P_2 + 2q_1^2 q_2$

$-\cancel{P_2} + \frac{\partial g}{\partial q_2}(q_1, q_2) = -\cancel{P_2} + 2q_1^2 q_2$

$g(q_1, q_2) = q_1^2 q_2^2 + h(q_1)$

I $\wedge$ II $\wedge$ III $\quad S = P_1 P_2 - q_2 P_2 + q_1 P_1 + q_1^2 q_2^2 + h(q_1)$

IV $\quad \frac{\partial S}{\partial q_1} = P_1 + 2q_1 q_2^2$

$\cancel{2q_1 q_2^2} + \cancel{P_1} + \frac{\partial h}{\partial q_1}(q_1) = \cancel{P_1} + \cancel{2q_1 q_2^2}$

$h(q_1) = c$ in particolare Spdg scelgo $c = 0$

$S = P_1 P_2 - q_2 P_2 + q_1 P_1 + q_1^2 q_2^2$

c] $\{f, g\} = \sum_{i=1}^{2} \left[ \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right] = \frac{\partial f}{\partial q_1}\frac{\partial g}{\partial p_1} - \frac{\partial f}{\partial p_1}\frac{\partial g}{\partial q_1} + \frac{\partial f}{\partial q_2}\frac{\partial g}{\partial p_2} - \frac{\partial f}{\partial p_2}\frac{\partial g}{\partial q_2}$

$\{q_i, q_j\} = 0 \qquad \{p_i, p_j\} = 0$

$\{q_i, p_j\} = \delta_{ij} \qquad \{p_i, q_j\} = -\delta_{ij}$

$\{Q_1, Q_2\} = \frac{\partial Q_1}{\partial q_1}\frac{\partial Q_2}{\partial p_1} - \frac{\partial Q_1}{\partial p_1}\frac{\partial Q_2}{\partial p_2} + \frac{\partial Q_1}{\partial q_2}\frac{\partial Q_2}{\partial p_2} - \frac{\partial Q_1}{\partial p_2}\frac{\partial Q_2}{\partial q_2}$

$= (1 + \cancel{4q_1 q_2}) \cdot 1 - \cancel{0(\dots)} + \cancel{(2q_1^2)} \cdot 0 - (-1)\cancel{(-4q_1 q_2 - 1)} = 0$

$$\{Q_1, P_1\} = \frac{\partial Q_1}{\partial q_1}\frac{\partial P_1}{\partial p_1} - \frac{\partial Q_1}{\partial p_1}\frac{\partial P_1}{\partial q_1} + \frac{\partial Q_1}{\partial q_2}\frac{\partial P_1}{\partial p_2} - \frac{\partial Q_1}{\partial p_2}\frac{\partial P_1}{\partial q_2} =$$

$$= (1 + \cancel{4q_1q_2})(1) - 0(\ldots) + (\cancel{2q_1^2})(0) - (-1)(\cancel{-4q_1q_2}) = 1$$

$$\{Q_i, Q_j\} = 0 \qquad \{P_i, P_j\} = 0 \qquad \{Q_i, P_j\} = \delta_{ij}$$

Ex.

$$\begin{cases} Q_1 = q_1 q_2 \\ Q_2 = q_1^2 - q_2^2 \\ P_1 = \dfrac{q_2 p_1 + q_1 p_2}{q_1^2 + q_2^2} \\ P_2 = \dfrac{1}{2}\,\dfrac{q_1 p_1 - q_2 p_2}{q_1^2 + q_2^2} \end{cases}$$

(Verificare che si conservino le parentesi di Poisson)

$$S(q_1, q_2, P_1, P_2)$$

$$p_1 = \frac{\partial S}{\partial q_1} \qquad p_2 = \frac{\partial S}{\partial q_2} \qquad Q_1 = \frac{\partial S}{\partial P_1} \qquad Q_2 = \frac{\partial S}{\partial P_2}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} \dfrac{q_2}{q_1^2 + q_2^2} & \dfrac{q_1}{q_1^2 + q_2^2} \\ \dfrac{1}{2}\dfrac{q_1}{q_1^2 + q_2^2} & -\dfrac{1}{2}\cdot\dfrac{q_2}{q_1^2 + q_2^2} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

$$\det A = -\frac{1}{2}\,\frac{1}{(q_1^2 + q_2^2)}$$

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = -2(q_1^2 + q_2^2) \begin{bmatrix} -\dfrac{1}{2}\dfrac{q_2}{q_1^2 + q_2^2} & -\dfrac{q_1}{q_1^2 + q_2^2} \\ -\dfrac{q_1}{2(q_1^2 + q_2^2)} & \dfrac{q_2}{q_2^2 + q_1^2} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$= \begin{bmatrix} q_2 & 2q_1 \\ q_1 & -2q_2 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\begin{cases} Q_1 = q_1 q_2 & = \dfrac{\partial S}{\partial P_1} & \text{I} \\ Q_2 = q_1^2 - q_2^2 & = \dfrac{\partial S}{\partial P_2} & \text{II} \\ p_1 = q_2 P_1 + 2 q_1 P_2 & = \dfrac{\partial S}{\partial q_1} & \text{III} \\ p_2 = q_1 P_1 - 2 q_2 P_2 & = \dfrac{\partial S}{\partial q_2} & \text{IV} \end{cases}$$

I $\quad \dfrac{\partial S}{\partial P_1} = q_1 q_2 \qquad S = q_1 q_2 P_1 + f(q_1, q_2, P_2)$

II $\quad \dfrac{\partial S}{\partial P_2} = q_1^2 - q_2^2$

$$\frac{\partial f}{\partial P_2} = q_1^2 - q_2^2$$

$$f = (q_1^2 - q_2^2) P_2 + h(q_1, q_2)$$

$$S = q_1 q_2 P_1 + (q_1^2 - q_2^2) P_2 + h(q_1, q_2)$$

III $\quad \dfrac{\partial S}{\partial q_1} = \cancel{q_2 P_1} + \cancel{2 q_1 P_2} + \dfrac{\partial h}{\partial q_1} = \cancel{q_2 P_1} + \cancel{2 q_1 P_2}$

$$h = g(q_2)$$

IV $\quad \dfrac{\partial S}{\partial q_2} = \cancel{q_1 P_1} - \cancel{2 q_2 P_2} + \dfrac{\partial g}{\partial q_2} = \cancel{q_1 P_1} - \cancel{2 q_2 P_2}$

$$S = q_1 q_2 P_1 + (q_1^2 - q_2^2) P_2 = Q_1(q_1, q_2) P_1 + Q_2(q_1, q_2) P_2$$

$$\begin{cases} Q_1 = Q_1(q_1, q_2) \\ Q_2 = Q_1(q_1, q_2) \\ p_1 = \dfrac{\partial Q_1}{\partial q_1} P_1 + \dfrac{\partial Q_2}{\partial q_1} P_2 \quad (*) \\ p_2 = \dfrac{\partial Q_2}{\partial q_1} P_1 + \dfrac{\partial Q_2}{\partial q_2} P_2 \end{cases}$$

TRASFORM. PUNTUALI

- $Q_1, Q_2$ devono dip. solo da $q_1, q_2$
- $P_1, P_2$ lin. omog. delle $p_i$

Se verifico che vale $(*)$, cioè

$$\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial Q_1}{\partial q_1} & \dfrac{\partial Q_2}{\partial q_1} \\ \dfrac{\partial Q_2}{\partial q_1} & \dfrac{\partial Q_2}{\partial q_2} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = J^T \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \quad (**)$$

allora $S = Q_1 P_1 + Q_2 P_2$ e la trasf. è canonica

Quindi

① $Q_1 = \dots$, $Q_2 = \dots$, $P_1 = \dots$, $P_2 = \dots$

② $p_1 = \dots$, $p_2 = \dots$

③ verifico $(**)$

Ex. $q_2 = \dfrac{\partial Q_1}{\partial q_1}$ $\quad 2q_1 = \dfrac{\partial Q_2}{\partial q_1}$

$q_1 = \dfrac{\partial Q_1}{\partial q_2}$ $\quad -2q_1 = \dfrac{\partial Q_2}{\partial q_2}$